



Two-layer integral model for calculating the evaporation rate from a liquid surface

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Abstract

An integral model is presented for predicting the evaporation rate from a single component liquid layer exposed to wind. This approach simplifies the solution of the convection–diffusion equation by averaging over the height of the concentration boundary-layer thickness. The model is valid for hydraulically smooth as well as rough surfaces and yields results which are in good agreement with those obtained by mathematically more rigorous methods. The new integral model is conceptually simpler and provides an explicit estimate of the concentration boundary-layer thickness. It allows a compact analytic solution for the evaporation rate with emphasis on the

Abbreviations: A , Area, e.g., area of a pool A_p ; b , Width of the pool; c , Concentration; c_f , Friction coefficient; c_p , Specific heat; c_s , Concentration at the saturation point; c_l , Specific heat of the pool; D , (Molecular) diffusion coefficient; $E(x)$, Mass-flow density flux of the evaporating fluid; $E_p(x)$, Mass-flow density flux of the evaporating fluid for high vapour pressure; $E_h(x)$, Heat-flow density flux from the wind to the evaporating fluid; \bar{E} , Mass-flow rate, averaged over the pool area; E' , Dimensionless mass-flow density flux of the evaporating fluid $E' = E/(c_s u_*^3)$; \bar{E}' , Dimensionless, area averaged mass-flow rate of the evaporating fluid $\bar{E}' = \bar{E}/(c_s u_*^3)$; k_h , Heat-transfer coefficient; k_m , Mass-transfer coefficient; K , Eddy viscosity; K' , Eddy diffusivity; L , Length, e.g., length of the pool in streamwise direction; L' , Dimensionless length of the pool $L' = L/z_0$; m_1 , Mass of the pool; p , Pressure; e.g., vapour pressure p_v ; \dot{q} , Heat-flow density flux; \dot{q}_s , Heat-flow density flux through radiation; r_e , Latent heat of vaporisation; Re , Reynolds number; Sc , (molecular) Schmidt number; St , Stanton number; St_m , Stanton number for mass transfer; T , Temperature; T_s , Temperature at the saturation point; T_a , Temperature of the ambient; u , Velocity of the windfield; u_1 , Velocity at the reference height z_1 ; u_* , Friction velocity; x , Coordinate in streamwise direction; x' , Dimensionless coordinate in streamwise direction $x' = x/z_0$; z , Coordinate normal to the liquid surface or pool; z' , Dimensionless coordinate normal to the liquid surface or pool $z' = z/z_1$; z_1 , Reference height; z_0 , Roughness length; z_{ov} , Vapour roughness length; *Greek letters*: β , Empirical function in the concentration profile Eq. (7); γ , $\gamma = (\sigma/\kappa n\beta)^{(1+2n)/n}$; Γ , $\Gamma = (\delta_c/z_1)^{1+2n}$; δ , Thickness of the velocity boundary layer; δ_c , Thickness of the concentration boundary layer; κ , von Kármán constant; μ , Dynamic viscosity; ν , Kinematic viscosity; ρ , Density; σ , Turbulent Schmidt number; τ , Shear stress

diffusion process in the laminar sublayer, accounted for by means of empirical wall functions.
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1. Introduction

Evaporation processes are frequently encountered in our natural environment [1] but they are also of interest in many technical applications, e.g. in the process industry. Analyses of hazards related to loss prevention also require accurate prediction of evaporation rates. The main objective herein will be the derivation of compact analytical expressions for predicting the evaporation rates, in view of the needs of the loss prevention analyst to produce quick but reliable estimates.

Past investigations of the evaporation phenomena in the free atmosphere include Webber and Brighton [2] and Böke and Hartwig [3]. In confined environments (e.g. tunnels), new problems occur, as discussed by Wren et al. [4].

The problem considered herein will be evaporation from wet surfaces, from pools or spills exposed to an airflow or the atmospheric wind. For volatile fluids with high vapour pressure, the evaporation rate will be high and the pool can cool down considerably unless it is deep. In this case, heat transfer from the ground can be important. For pools consisting of more than one component, concentration gradients can be important [5]. Here, only single-component pools will be considered.

1.1. General outline of the present contribution

The present study will be restricted to the problem of pool evaporation in the wind field of a neutral atmosphere. The problem will be two-dimensional with an evaporating surface of finite length in the flow direction but of infinite extent transverse to the flow (see Fig. 1).

In a general overview (Section 2.1), the two representative models of Sutton [6] and Brighton [7] for solving the convection–diffusion equation will be presented. The underlying assumptions and simplifications will be discussed, in particular, the improvements of Brighton's model with respect to that of Sutton.

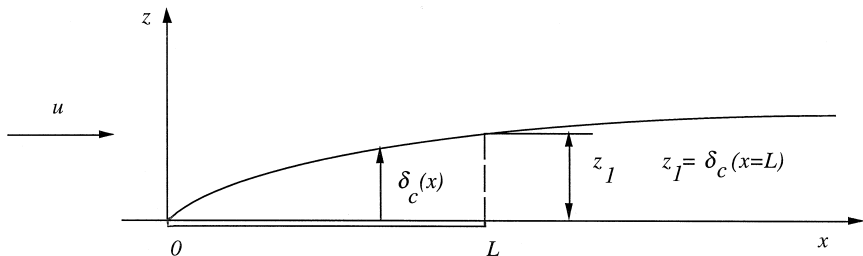


Fig. 1. Concentration boundary layer above a liquid pool in the wind.

In subsequent sections, the integral methods, familiar to engineers interested in mass and heat transfer, following the lead of von Kármán or Pohlhausen, are presented. Here, the equations are satisfied only by averaging over the thickness of the concentration boundary layer [8]. In order to demonstrate the main features of the integral method, the integral formulation and the corresponding solution of Sutton's model will be compared, using the same physical assumptions as in the original model of Sutton.

A new method, based on the same physical assumptions as used in the method of Brighton [7], will be presented in Section 2.3. It is conceptually simpler and more intuitive than the method of Brighton and makes use of the concept of boundary-layer integral methods. As in similar applications in the past, it will be demonstrated that the method offers good accuracy with little computational effort. The simple analytic expressions for the mass-flow rate of the evaporating fluid will be useful for safety studies. The present method allows also an explicit but approximate solution for the thickness of the concentration boundary layer not derived in the mathematically more rigorous approaches of Sutton or Brighton. The power profiles, approximating the logarithmic velocity distribution, will be improved based on the analytic estimate of the concentration boundary-layer thickness.

2. Evaporation from liquid surface layers and pools

Fluids, with boiling points well above the ambient temperature, are in a liquid state under standard conditions (e.g. water, diesel fuel, etc.) and their evaporation rates are small due to their small vapour pressures. The evaporation rate and how it changes with time is controlled by a mass-transfer process.

2.1. General overview

The solution of Sutton [6] as well as that of Brighton [7] of the convection–diffusion problem will be presented in this section. The improvements of the approach of Brighton with respect to that of Sutton will be outlined. The physical basis of both models will be described as it is used also in the formulation of the integral model proposed herein.

The concentration field above an evaporating pool can be described by means of a convection–diffusion equation. When simplified using the boundary-layer approximation, it takes the form [9]:

$$u(z) \frac{\partial c}{\partial x} = \frac{\partial}{\partial z} \left[K'(z) \frac{\partial c}{\partial z} \right]. \quad (1)$$

In what follows, a rectangular pool of unit width ($b = 1$) and of length L in the direction of the wind is considered. The gradients of velocity vanish in the flow direction because the wind field is fully developed: $\partial u / \partial x = 0$.

The solution given by Sutton [6] will be discussed first. The velocity field is approximated by means of a power law: $u = u_1(z/z_1)^n$, where u_1 represents the velocity at a reference height z_1 . Sutton presented his solution for a general exponent n , but

specified $n = 1/7$ for neutral atmospheric boundary layers. A justification for this value of the exponent is given, e.g. by Schlichting [8]. The exponent $n = 1/7$ is hereby derived from the resistance formula of Blasius for fully developed turbulent pipe flow. This is the same value used for flat plate flows [8]. Due to the similarity of the turbulent flow in a pipe and over a flat plate, the same power law is used for the velocity profiles in both cases [8].

The mass-flow rate of the evaporating liquid $E = -K'\partial c/\partial z$ depends on the eddy diffusivity K' describing the concentration field. The eddy diffusivity K' is proportional to the eddy viscosity K , i.e. $K' = (1/\sigma)K$, with the turbulent Schmidt number denoted σ .

Sutton used von Kármán's formulation of the eddy viscosity with mixing length $l = \kappa \left| \frac{\partial u/\partial z}{\partial^2 u/\partial z^2} \right|$. Applying the 'statistical theory method', outlined in [6], Sutton obtained for the eddy viscosity:

$$K = 0.1386 \nu^{1/4} u_1^{3/4} z^{6/7} z_1^{-3/28}, \quad (2)$$

with $n = 1/7$ in the power law for the velocity field.

The shear stress τ is accordingly:

$$\frac{\tau}{\rho} = 0.020 \left(\frac{\nu}{u_1 z_1} \right)^{1/4} u_1^2,$$

when von Kármán constant $\kappa = 0.40$ and a turbulent Schmidt number $\sigma = 1$ are used [6].

The classical value of the coefficient from Prandtl/Blasius is 0.0225 for hydraulically smooth surfaces, remembered here for comparison [6,8]. The solution of Sutton is based on the method of singularities, and the rectangular pool is modeled by superposition of line sources of infinite length transverse to the wind. For the area-averaged mass-flow rate of the evaporating substance, Sutton [6] obtained:

$$\bar{E} = 0.0277 c_s \text{Re}_1^{-2/9} z_1^{1/9} u_1 L^{-1/9}, \quad (3)$$

where c_s is the vapour concentration for saturated vapour and $\text{Re}_1 = u_1 z_1/\nu$. The frequent use of Sutton's model is due mainly to the compact analytic expression of \bar{E} .

Some weak points of the model have been pointed out by Brighton [7]. One objection is that the turbulent mass transfer is not linked to the molecular diffusion process, which plays a major role in the laminar sublayer close to the evaporating surface. Moreover, \bar{E} is not formulated in dimensionless form with respect to the relevant similarity parameters Re and Sc . Finally, parameters, such as the friction velocity u_* and the roughness length z_o , that define the inner structure of the boundary layer, are not taken into account. As an improvement, Brighton makes use of the logarithmic velocity distribution, defined by u_* and z_o , for the neutral boundary layer:

$$u(z) = \frac{u_*}{\kappa} \ln \left(\frac{z}{z_o} \right). \quad (4)$$

corresponding to a linear eddy-viscosity profile $K = \kappa u_* z$ (see Ref. [10] or [8]). The logarithmic velocity distribution Eq. (4) is not well suited for an analytic solution of Eq. (1) and it is approximated by means of a power law:

$$u = u_1 \left(\frac{z}{z_1} \right)^n \tag{5}$$

Formally, this is the same as that used by Sutton, but in order to obtain an optimal approximation in the concentration boundary layer, the reference height z_1 should be related now to the thickness of the concentration boundary layer. Therefore, equality of the velocity u and of the gradient du/dz is required at location z_1 for both profiles Eqs. (4) and (5). Then u_1 and n can be derived in a subsequent step. An estimate of z_1 is given by the theory of line sources [10].

Brighton used a linear eddy-diffusivity profile $K' = K/\sigma = \kappa u_* z/\sigma$, yielding the logarithmic velocity distribution Eq. (4). By substituting K' and the velocity distribution Eq. (5) into the convection–diffusion Eq. (1), he obtained the following linear differential equation:

$$u_1 \left(\frac{z}{z_1} \right)^n \frac{\partial c}{\partial x} = \frac{\partial}{\partial z} \left[\frac{\kappa u_* z}{\sigma} \frac{\partial c}{\partial z} \right] \tag{6}$$

Brighton solved this equation by means of Laplace transforms. He matched the concentration profile, obtained by solving Eq. (6), to a wall profile that introduces the effects of molecular diffusivity in a small region close to the liquid surface. By using a linear eddy-diffusivity profile (see, e.g. Ref. [10]) $E = -K'\partial c/\partial z = -(\kappa/\sigma)\partial c/\partial z = -(\kappa\mu_* z/\sigma)\partial c/\partial z$ can be integrated to obtain such a wall profile:

$$\frac{c}{c_s} = 1 - \frac{E}{c_s u_*} \left[\frac{\sigma}{\kappa} \ln \left(\frac{z}{z_0} \right) + \beta \right] \tag{7}$$

The term β can be considered as an integration constant and accounts for molecular diffusion processes in the laminar sublayer. β stems from empirical correlations and has different forms depending on the properties of the surface. This can be either hydrodynamically smooth or rough. An overview of the different forms of β , including a discussion of the physical background, is given by Brutsaert [1]. A ‘heuristic’ representation of the two-layer model for the velocity and the concentration field, with definitions used in the present work, is given in Appendix A.

Brighton [7] takes for the hydrodynamically rough surfaces:

$$\beta = 7.3\text{Re}_0^{1/4} \text{Sc}^{1/2} - 5\sigma \text{ where } \text{Re}_0 = \frac{u_* z_0}{\nu},$$

and for the hydrodynamically smooth surfaces:

$$\beta = (3.85\text{Sc}^{1/3} - 1.3)^2 + \frac{\sigma}{\kappa} \ln(\text{Sc}) + \frac{\sigma}{\kappa} \ln(0.135).$$

The profile given by Eq. (7) is universal in the vicinity of the wall (for Schmidt numbers larger than 0.5). It includes the molecular diffusion process in the laminar

sublayer and the turbulent diffusion described by K' . In the following, it will be denoted the 'wall profile' in accord with Brighton [7].

The effective roughness for hydrodynamically smooth surfaces is $z_o = 0.13 \nu/u_*$; whereas only crude estimates are available for hydrodynamically rough surfaces (cf. Brutsaert [1]). The integrals occurring in the algorithm of Brighton can be solved by means of developments in series of the appropriate variables in the transformed space. The results of this mathematically rigorous integration of the convection–diffusion equation will be used for comparison with the results of the approximate integral approach, which is developed in the present contribution.

2.2. General features of the integral procedures

This section deals with the integral approach of the convection–diffusion equation. Integral methods, in general, are approximate methods where the boundary-layer equations are satisfied only on an average extended over the thickness of the boundary layer. These methods, which can be attributed to von Kármán and Pohlhausen, are well documented in Schlichting [8]. Techniques for approximating the profiles in a velocity or concentration boundary layer are addressed and their limitations are discussed in this reference.

On integrating Eq. (1) over the height δ_c , one obtains:

$$\int_0^{\delta_c} \frac{\partial}{\partial x}(uc) dz = \left[K' \frac{\partial c}{\partial z} \right]_0^{\delta_c} = E(x). \quad (8a)$$

Because the diffusive transport is much larger at the liquid surface than at the outer edge δ_c of the concentration boundary layer, whose thickness increases continuously, one obtains for the RHS of Eq. (8a):

$$E(x) = -K' \frac{\partial c}{\partial z} \Big|_{z=0} = -\frac{K}{\sigma} \frac{\partial c}{\partial z} \Big|_{z=0}. \quad (8b)$$

The concentration vanishes at the outer edge δ_c of the concentration boundary layer, i.e., $c(\delta_c) = c_\infty = 0$ (or in case of $c_\infty \neq 0$: $c'(\delta_c) = 0$ where $c' = c - c_\infty$). Such approximate formulations of the boundary conditions are normally used in the integral approach in boundary-layer theory [8]. With $c(\delta_c) = 0$, the differential operator can be put in front of the integral to obtain:

$$\frac{d}{dx} \int_0^{\delta_c} (uc) dz = E(x). \quad (9)$$

The integration of Eq. (9) over the length L of the pool gives a mass-flow rate per unit width, which is averaged over the length of the pool:

$$\bar{E} = \frac{1}{L} \int_0^L E(x) dx = \frac{1}{L} \int_0^{\delta_c(L)} (uc) dz. \quad (10)$$

Fig. 1 allows a straightforward interpretation of this formula: Eq. (10) represents the convective mass-flow rate through the vertical plane at location $x = L$.

2.2.1. Integral formulation and the corresponding solution of Sutton’s model

The integration of the height-averaged differential equation for the convection-diffusion problem, Eq. (9), is the main objective of this section. Agreement between the integral approach and Sutton’s work will be demonstrated. Basic assumptions related to the physics of Sutton’s model, as discussed in Section 2.1, are therefore also used in the present integral formulation. The eddy viscosity Eq. (2), used by Sutton, is introduced in Eq. (9). In the integral approach, the profile for the velocity u is approximated using the power law of Eq. (5) already used by Sutton. The one-seventh power law for the velocity profile suggests also the form of the concentration profile, which fulfils the following boundary conditions at the surface and at the outer edge, i.e., $c(z = 0) = c_s$, resp. $c(z = \delta_c) = 0$:

$$c = c_s \left[1 - \left(\frac{z}{\delta_c} \right)^{1/7} \right]. \tag{11}$$

The thickness of the concentration boundary layer, that can vary in the streamwise direction, i.e., $\delta_c = \delta_c(x)$, is still unknown. Here, similarity concepts in boundary-layer theory are used by choosing profiles where the ratio (z/δ_c) (with $\delta_c = \delta_c(x)$) can be considered as a similarity variable η . Eq. (11) is only an approximation of the real concentration profile in the concentration boundary layer up to the upper boundary $z = \delta_c$. The good agreement of the area-averaged mass-flow rate with Sutton’s result demonstrates the validity of the profile approximation, in particular the choice of the one-seventh power in Eq. (11). For a documentation of the integral approach, including a discussion of different assumptions for the boundary-layer profiles, it is referred to Ref. [8].

The mass-flow density flux $E(x)$, i.e. Eq. (8b), can be obtained with the help of the eddy viscosity, Eq. (2), and the concentration profile, Eq. (11):

$$E(x) = 0.020 \sigma^{-1} c_s \nu^{1/4} u_1^{3/4} z_1^{-3/28} \delta_c^{-1/7}, \tag{12}$$

where $E(x)$ does not depend on z .

With the profile assumptions Eqs. (5) and (11), the integral on the LHS of Eq. (9) can be evaluated. Substituting $E(x)$ from Eq. (12) in the RHS of Eq. (9), a differential equation for the unknown concentration boundary-layer thickness $\delta_c(x)$ is obtained. The solution is:

$$\delta_c(x) = 0.318 \sigma^{-7/9} u_1^{-7/36} z_1^{1/36} \nu^{7/36} x^{7/9}. \tag{13}$$

With this result, the mass-flow density flux $E(x)$, Eq. (12), is also known. The integral in Eq. (10) can be evaluated to yield the area-averaged mass-flow rate:

$$\bar{E} = \frac{1}{L} \int_0^L E(x) dx = 0.0262 \sigma^{-8/9} c_s \text{Re}_1^{-2/9} u_1 z_1^{1/9} L^{-1/9}. \tag{14}$$

The coefficient 0.0262 is quite close to Sutton’s value 0.0277. The numerical value obtained depends on the von Kármán constant, the value used here is $\kappa = 0.40$. For the turbulent Schmidt number, Sutton chose the value $\sigma \cong 1$. In contrast to Sutton’s model, the solution obtained from the integral formulation yields also an estimate of the thickness of the concentration boundary layer $\delta_c(x)$ with elementary algebra.

2.3. Present integral model

The approximate profiles for velocity and concentration, which are required for the integration of the height-averaged differential equation for the convection-diffusion problem, Eq. (9), are identical with those used by Brighton [7]. The profile for velocity is given by Eq. (4) and the profile for concentration by Eq. (7).

The term β is discussed in Section 2.1. Since the concentration profile includes the molecular diffusion processes in the laminar sublayer as well as the turbulent diffusion described by K' (Section 2.1), it represents a reasonable approximation for the whole height of the concentration boundary layer. The boundary conditions are accordingly $c = c_s$ at the liquid surface or inner edge and $c(z = \delta_c) = c_\infty = 0$ at the outer edge of the concentration boundary layer. If the evaporating component is present in the outer field, $c = c_\infty \neq 0$. In this case, a new variable $c' = c - c_\infty$ is defined.

In the 'exact' analytic solution of Brighton [7], the logarithmic velocity profile was approximated by a power law given in Eq. (5).

The value of the reference height z_1 should be related to the thickness of the concentration boundary layer. It will be discussed later. The values n and u_1 are determined by the requirement that the power law should be a suitable approximation to the logarithmic velocity profile at height z_1 :

$$\frac{1}{n} = \ln \left(\frac{z_1}{z_0} \right), \quad (15)$$

and:

$$\left(\frac{u_1}{u_*} \right) = \frac{1}{\kappa n}. \quad (16)$$

Eq. (16) is substituted into Eq. (5) to obtain:

$$\left(\frac{u}{u_*} \right) = \frac{1}{\kappa n} \left(\frac{z}{z_1} \right)^n. \quad (17)$$

Since Eq. (7) is not well suited for an analytic integration over the relevant height of the convection-diffusion equation, an approximation is also required for the concentration field. By combining Eq. (4) with Eq. (7), one obtains a concentration profile which is consistent with the velocity profile:

$$\left(\frac{c}{c_s} \right) = 1 - \frac{E}{c_s u_*} \left[\sigma \left(\frac{u}{u_*} \right) + \beta \right], \quad (18)$$

and with the approximation Eq. (17) for u :

$$\left(\frac{c}{c_s} \right) = 1 - E' \left[\sigma \frac{1}{\kappa n} \left(\frac{z}{z_1} \right)^n + \beta \right] \text{ where } E' = \frac{E}{c_s u_*}. \quad (19)$$

In order to obtain a suitable approximation for the concentration boundary layer, z_1 is set equal to the thickness δ_c of the concentration boundary layer at the end of the pool.

$$z_1 = \delta_c (x = L).$$

(The mean value $z_1 = 0.5\delta_c$ ($x = L$) would be more accurate, but the effect on the final numerical result is small).

The value of n is unknown a priori, but the value $n = 1/7$, from the power law for the velocity, can be taken as a first approximation, as δ_c is unknown in the first iteration step. A better value of n can be obtained from Eq. (15) in a subsequent step.

The boundary condition $c = c_\infty = 0$ is valid at the outer edge $z = \delta_c$ of the concentration boundary layer. On using this condition in Eq. (19), which is solved for E' , one obtains:

$$E' = \left[\frac{\sigma}{\kappa n} \left(\frac{\delta_c}{z_1} \right)^n + \beta \right]^{-1} \tag{20}$$

The boundary condition $c(z=0) = c_s$ at the liquid surface or inner edge of the boundary layer is not correctly accounted for. In effect, it is shifted from the liquid surface $z = 0$ to a finite but small z -value, producing a small error. This error in the concentration profile near the wall has little effect on the final result for two reasons: (a) the approximation of the logarithmic velocity profile by means of a power law is not very accurate in the vicinity of the wall anyway (likewise true for the ‘exact’ solution of Brighton), and (b) the convective contribution in the neighbourhood of the wall is negligible. Also, the mismatch of the concentration gradient $\partial c/\partial z$ at the outer edge of the concentration boundary layer $z = \delta_c$, which cannot be correctly accounted for, is a weak point here, as in other integral boundary-layer solutions using power-law profiles. The successful comparison of the results obtained, by the present model with those of Brighton, supports the validity of the assumptions made.

The convection–diffusion equation in integral form, Eq. (9), can be written in the non-dimensional form:

$$\left(\frac{z_1}{z_o} \right) \frac{d}{dx'} \int_0^{(\delta_c/z_1)} \left(\frac{u}{u_*} \right) \left(\frac{c}{c_s} \right) dz' = E', \tag{21}$$

when the dimensionless variables $x' = x/z_o$ and $z' = z/z_1$ are introduced. The coordinate x is nondimensionalized by the roughness length z_o , which is assumed constant for the problem considered, in accord with Brighton. The definition of z' is also in accord with Brighton; z_1 corresponds approximately to the thickness of the concentration boundary layer.

The ratio (z_1/z_o) , from Eq. (15), can be substituted into Eq. (21) to give:

$$e^{1/n} \frac{d}{dx'} \int_0^{(\delta_c/z_1)} \left(\frac{u}{u_*} \right) \left(\frac{c}{c_s} \right) dz' = E'. \tag{22}$$

The profiles Eqs. (17) and (19) are now substituted into the integrand of Eq. (22). Upon evaluation, one obtains:

$$\frac{1}{C_1} \frac{d}{dx'} \left[E' \left(\frac{\delta_c}{z_1} \right)^{1+2n} \right] = E' \text{ with } C_1 = \frac{\kappa^2 n(1+n)(1+2n)}{\sigma e^{1/n}}. \tag{23}$$

The differential equation Eq. (23) and the algebraic equation Eq. (20) can be solved

for the unknown dimensionless mass-flow density flux E' and the boundary-layer thickness δ_c . The solution procedure, which is based purely on algebraic operations, can be found in Appendix B.

The solution for the thickness of the concentration boundary layer is:

$$\left(\frac{\delta_c}{z_1}\right) = (C_1 C_2 x')^{\frac{1}{1+2n}} \text{ with } C_2 = 1 + \frac{1}{2} \frac{n}{1+n}. \tag{24}$$

When this result is substituted into Eq. (20), the mass-flow density flux E' can be obtained:

$$E' = \left[\beta + \frac{\sigma}{\kappa n} (C_1 C_2 x')^{\frac{n}{1+2n}} \right]^{-1}. \tag{25}$$

The concentration boundary-layer thickness, normalized by the known roughness length z_o , is:

$$\left(\frac{\delta_c}{z_o}\right) = \left(\frac{z_1}{z_o}\right) \left(\frac{\delta_c}{z_1}\right) = e^{1/n} (C_1 C_2 x')^{\frac{1}{1+2n}}, \tag{26}$$

where (z_1/z_o) is provided by Eq. (15) and (δ_c/z_1) by Eq. (24).

Now the estimate of the power n can be improved. The first approximation of n will be denoted by n_a . The logarithmic profile Eq. (4) for the velocity field is approximated by the power law of Eq. (5). In order to describe the convective transport of the evaporated fluid adequately, the power-law approximation should be valid over the height of the concentration boundary layer. This is achieved when z_1 corresponds approximately to the thickness of the concentration boundary layer at the downstream edge of the pool δ_c ($x = L$).

An improved value of z_1 corresponding to δ_c ($x = L$), denoted $z_{1\text{new}}$, can be obtained when x' is set equal to L in Eq. (26), i.e.,

$$\left(\frac{z_{1\text{new}}}{z_o}\right) = \left(\frac{\delta_c(x=L)}{z_o}\right) = e^{1/n_a} (C_1 C_2 L)^{\frac{1}{1+2n_a}}. \tag{27}$$

When $(z_{1\text{new}}/z_o)$ is substituted into Eq. (15), an improved value of n is obtained:

$$\frac{1}{n} = \ln\left(\frac{z_{1\text{new}}}{z_o}\right) = \frac{1}{n_a} + \frac{1}{1+2n_a} \ln(C_1 C_2 L) \text{ with } n_a = 1/7. \tag{28}$$

The values of n at both extremes of the range considered here are $n(L = 10^3) \cong 1/4.5$ and $n(L = 10^7) \cong 1/11.6$. The calculations documented in Figs. 2 and 3 are based on the values of the power n given by Eq. (28) (one iteration).

The dimensionless mass-flow rate, averaged over the pool area, is:

$$\bar{E}' = \frac{1}{L} \int_0^L E' dx' = \frac{1}{L C_1} \left[E' \left(\frac{\delta_c}{z_1}\right)^{1+2n} \right]_{x'=L}, \tag{29}$$

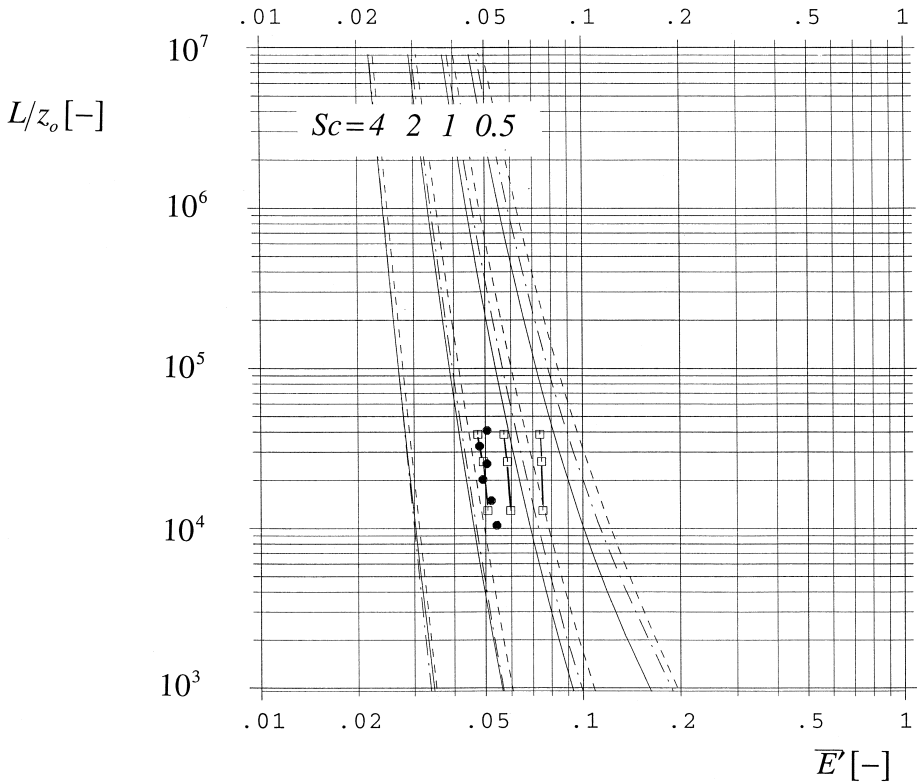


Fig. 2. Area-averaged dimensionless mass-flow rate for the evaporation above a hydrodynamically smooth surface (Parameter $Sc = 4; 2; 1; 0.5$; from the left to the right) ‘exact solution’ of Brighton [7]: ---, numerical solution by means of the integral formulation: - · -, approximate analytic solution of the integral formulation: ——— (Eq. (30)), numerical values of Reijnhart et al. [12]: □ ——— □, (Parameter $Sc = 1.7; 1.4; 1.0$; from the left to the right), experimental values of Reijnhart and Rose [11] ($Sc = 1.7$): ● ●.

where the integrand E' is given by the LHS of Eq. (23). The initial condition that the concentration boundary layer starts at the upstream edge of the pool, i.e. $\delta_c(x' = 0) = 0$, has been used here.

The dimensionless area-averaged mass-flow rate Eq. (29) can be rewritten, when Eqs. (24) and (25) are substituted into Eq. (29):

$$\bar{E}' = \frac{C_2}{\beta + \frac{\sigma}{\kappa n} (C_1 C_2 L')^{\frac{n}{1+2n}}}. \tag{30}$$

with the power n given by Eq. (28).

The mass-flow rate, averaged over the pool area is now $\bar{E} = \bar{E}' u_* c_s = k_m c_s$ where k_m is the mass-transfer coefficient.

The area-averaged dimensionless mass-flow rate, as obtained by means of the compact expression Eq. (30), can be compared to the area-averaged dimensionless

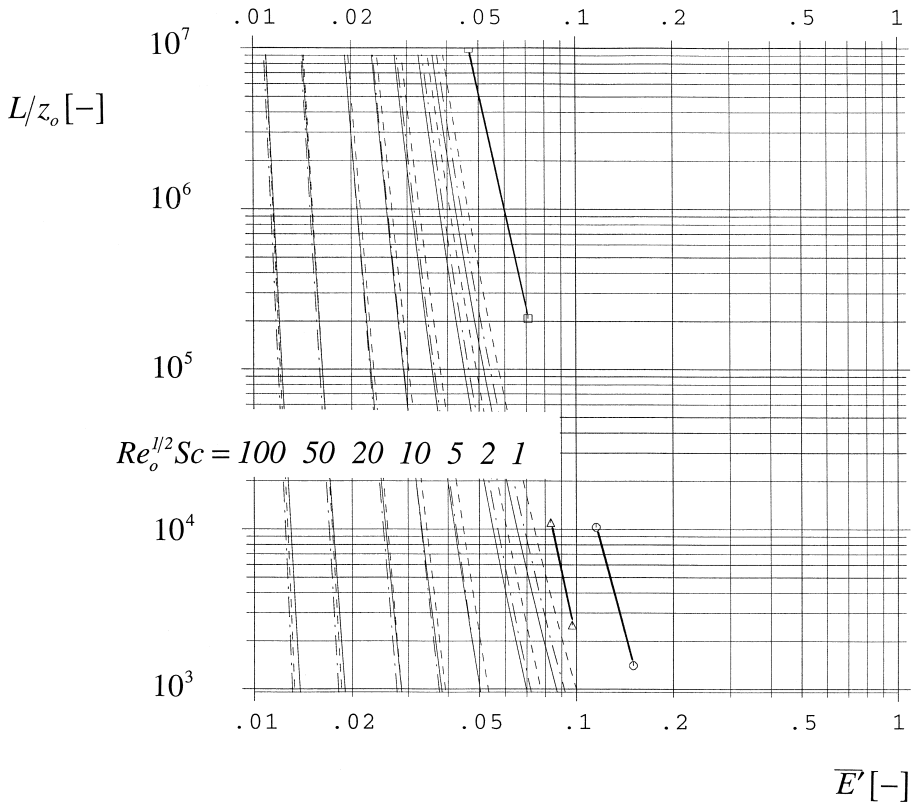


Fig. 3. Area-averaged dimensionless mass-flow rate for the evaporation above a hydrodynamically rough surface (Parameter $Re_o^{1/2} Sc = 100; 50; 20; 10; 5; 2; 1$; from the left to the right) 'exact solution' of Brighton [7]: ---, numerical solution by means of the integral formulation: - · -, approximate analytic solution of the integral formulation: ——— (Eq. (30)), numerical results of Reijnhart et al. [12]: Δ ——— Δ , correlations of Brutsaert [1] (evaporation pans): \circ ——— \circ , correlations of Brutsaert [1] (lakes): \square ——— \square .

mass-flow rate, determined by means of Brighton's model, for the hydrodynamically smooth case (cf. Fig. 2) as well as for the hydrodynamically rough case (cf. Fig. 3). The discrepancy is always less than 10%. In case of a fixed power $n = n_a = 1/7$, instead of the improved value given by Eq. (28), the discrepancy can exceed 20% (for $Sc = 0.5$ (hydrodynamically smooth surface) and for $Re_o^{1/2} Sc = 1$ (hydrodynamically rough surface)).

A comparison with the most recent experimental and theoretical data for evaporation of single component liquids, as already presented by Brighton, is shown in order to give an idea of the accuracy of the present model. The main features of the different approaches will be resumed here. The experimental data of Reijnhart and Rose [11] (Fig. 2: ●●●, $Sc = 1.7$) for smooth surfaces were obtained in a wind tunnel. They provided numerical values for u_* and achieved a fully developed logarithmic boundary

layer, two prerequisites for a reasonable comparison to the present model or that of Brighton. Both requirements are generally not met in most experiments reported in the literature. The numerical solution of Reijnhart et al. [12] (Fig. 2: □ — □, Sc = 1.0; 1.4 and 1.7) for solving the convection–diffusion equation, with allowance for molecular effects near the wall, is based on physical arguments similar to those of the present model.

The numerical results from Reijnhart and Rose [11], using the Reijnhart et al. [12] model for rough walls, with no allowance for molecular diffusion at the wall, correspond to the following value of the parameter $Re_o^{1/2}Sc = 0.34$ (Fig. 3: △ — △). The correlations for large evaporating surfaces, like lakes, (Fig. 3: □ — □) and for small evaporation pans (Fig. 3: o — o), given by Brutsaert [1], are based on his recommended value $z_o = 2.28 \times 10^{-4}$ m. The discrepancies observed have to be considered in the light of the scatter of the original data used by Brutsaert [1]. The existence of a roughness change from the shore to the evaporating surface has also been ignored in the present model. In order to render a comparison possible, Brighton assumed a typical wind speed of 5 m/s at height 10 m, yielding $Re_o = 2.8$ and $Re_o^{1/2}Sc = 1.0$ under these conditions.

When the vapour pressure of the fluid is high, the normal velocity at the pool surface is considerable and the convective contribution to the vertical mass transfer cannot be neglected. With the boundary condition, stipulating that no air penetrates into the pool (one-sided diffusion), the dimensionless mass-flow rate at high vapour pressure (cf. Refs. [7,13]) can be obtained:

$$E'_p = -E' \frac{p}{p_v} \ln \left(1 - \frac{p_v}{p} \right),$$

where p is the total pressure and p_v is the vapour pressure.

The area-averaged dimensionless mass-flow rate can be written accordingly:

$$\bar{E}'_p = -\bar{E}' \frac{p}{p_v} \ln \left(1 - \frac{p_v}{p} \right). \tag{31}$$

where \bar{E}' is given by Eq. (30). The area-averaged mass-flow rate \bar{E} is:

$$\bar{E}'_p = c_s u_* \bar{E}' = c_s k_m. \tag{32}$$

2.3.1. Dependence on pool length (comparison with Sutton)

The dependence of the area-averaged mass-flow rate \bar{E} on the pool length L will be analysed next. Eq. (30) yields:

$$\bar{E} = c_s u_* \bar{E}' = c_s u_* \frac{C_2}{\beta + \frac{\sigma}{\kappa n} (C_1 C_2 L)^{\frac{n}{1+2n}}}. \tag{33}$$

β in the denominator can be neglected if the length of the pool $L = \frac{L}{z_0} \gg \frac{1}{C_1 C_2} \left(\frac{\beta \kappa n}{\sigma} \right)^{\frac{1+2n}{n}}$. It follows:

$$\bar{E} \approx (L)^{\frac{-n}{1+2n}}. \quad (34)$$

With $n = 1/7$, Sutton's well known dependence on pool length is recovered as a particular case (see Eq. (14)). It follows that the validity of Eq. (34), adopted by Mackay and Matsugu [14] and many other authors as a basis for empirical or numerical models, is restricted. The neglect of β would be equivalent to the neglect of the laminar sublayer.

2.4. Heat transfer

A convection–diffusion equation for the temperature field, analogous to Eq. (1), can be formulated in case of heat exchange with the ambient [9]. The solution Eq. (30) can be adopted accordingly for the area–averaged dimensionless heat-flux:

$$\bar{E}'_h = \frac{C_2}{\beta_h + \frac{\sigma_h}{\kappa n} (C_1 C_2 L)^{\frac{n}{1+2n}}}, \quad (35)$$

and the dimensional area–averaged heat-flux:

$$\bar{E}_h = \rho c_p u_* (T_\infty - T_s) \bar{E}'_h = \frac{C_2}{\beta_h + \frac{\sigma_h}{\kappa n} (C_1 C_2 L)^{\frac{n}{1+2n}}} \rho c_p u_* (T_\infty - T_s), \quad (36)$$

or, in compact form, in terms of the heat-transfer coefficient k_h :

$$\bar{E}_h = k_h (T_\infty - T_s). \quad (37)$$

β_h corresponds to the β of Eq. (7) where the Schmidt number Sc has to be replaced by the Prandtl number Pr . There is still some discussion about the best value of the turbulent Prandtl number σ_h , with proposed values ranging from 0.74 to 1.0 [1]. A value for the turbulent Prandtl number $\sigma_h = 0.85$, which is consistent with the turbulent Schmidt number used by Brighton, is considered to be reasonable.

2.5. Heat balance of the pool

A general formulation of the heat balance of a pool is given by:

$$m_\ell c_\ell \frac{dT_s}{dt} = -E_p r_e + E_h + \dot{q} A_p + \dot{q}_s A_p, \quad (38)$$

where m_ℓ is the mass of the pool, c_ℓ its specific heat, $E_p r_e$ the loss of heat due to evaporation, E_h the heat addition from the wind or airflow, $\dot{q} A_p$ the heat addition from the ground and $\dot{q}_s A_p$ the heat addition by radiation. The system attains an equilibrium

state when the heat needed for evaporation is balanced by the heat gained from the airflow. In this case, the heat balance of the pool Eq. (38) yields, in terms of the area-averaged quantities:

$$0 = -\bar{E}_p r_e + \bar{E}_h. \tag{39}$$

The expressions in Eq. (32) for \bar{E}_p and Eq. (37) for \bar{E}_h can be substituted into Eq. (39) to obtain:

$$\frac{1}{r_e} \frac{k_h}{k_m} (T_\infty - T_s) = c_s. \tag{40}$$

It has been assumed here that no radiation is present and that the pool temperature is equal to the temperature of the ground. When the vapour pressure is low and when the molecular Schmidt and Prandtl numbers are equal (including the case when the turbulent Schmidt and Prandtl numbers are assumed to be equal, i.e. $\sigma = \sigma_h$), a well-known formula, e.g., used in psychometry, is reproduced [15]:

$$\frac{\rho c_p}{r_e} (T_\infty - T_s) = c_s. \tag{41}$$

When the airflow over the pool is purely laminar, one obtains for nonequal Schmidt and Prandtl numbers:

$$\frac{\rho c_p}{r_e} \left(\frac{Sc}{Pr} \right)^{2/3} (T_\infty - T_s) = c_s. \tag{42}$$

This expression, often generalized to the turbulent case, does not account for the laminar sublayer, in contrast to Eq. (40a).

Both the left and the right hand side of Eq. (40) can be represented graphically as functions of the temperature T_s . The temperature at the equilibrium point is found by the intersection of the (near linear) contribution of the left hand side with the concentration at the saturation point $c_s = c_s(T_s)$ of the right hand side. It is noted that c_s is large for volatile fluids with high vapour pressure, so that the pool can cool down considerably. In this case, heat transfer from the ground becomes important. The temperature, the vapour pressure and the evaporation rate will depend strongly on time when the initial temperature of the pool is near ambient or higher (Wren et al. [4]).

2.6. Summary of the procedure

When Eq. (30) is substituted into Eq. (31), which in turn is substituted into Eq. (32), the calculation of the area-averaged mass-flow rate can be reduced to the evaluation of the two expressions Eq. (43) and Eq. (28):

$$\bar{E}_p = -c_s u_* \frac{p}{p_v} \ln \left(1 - \frac{p_v}{p} \right) \frac{C_2}{\beta + \frac{\sigma}{\kappa n} (C_1 C_2 L)^{\frac{n}{1+2n}}} = k_m c_s \quad (L = L/z_o), \tag{43}$$

with:

$$C_1 = \frac{\kappa^2 n(1+n)(1+2n)}{\sigma e^{1/n}} \quad C_2 = 1 + \frac{1}{2} \frac{n}{1+n}.$$

The power $n = n_{i+1}$ is given by Eq. (28a), which is written in a form suitable for an iterative procedure:

$$\frac{1}{n_{i+1}} = \frac{1}{n_i} + \frac{1}{1+2n_i} \ln(C_1 C_2 L), \quad (28a)$$

with the starting value $n_i = n_a = 1/7$. Numerical experiments show that one iteration is sufficient.

The total mass-flow rate is $E_p = A\bar{E}_p$ with the surface of the pool $A = Lb$ and its length L in the streamwise direction.

The calculation of the heat-flux averaged over the surface area of the evaporating pool can be calculated in an analogous manner with Eq. (36):

$$\begin{aligned} \bar{E}_h &= \rho c_p u_* (T_\infty - T_s) \bar{E}'_h \\ &= \frac{C_2}{\beta_h + \frac{\sigma_h}{\kappa n} (C_1 C_2 L)^{\frac{1}{1+2n}}} \rho c_p u_* (T_\infty - T_s) = k_h (T_\infty - T_s), \end{aligned} \quad (36a)$$

with $n = n_{im}$ given by Eq. (28a).

When the system has attained an equilibrium state, the temperature at the saturation point T_s is given by Eq. (40):

$$\frac{1}{r_e} \frac{k_h}{k_m} (T_\infty - T_s) = c_s(T_s). \quad (40a)$$

3. Discussion

The present work is focused on an integral model for solving the height-averaged convection–diffusion equation. One main objective is to develop a simple solution of the convection–diffusion problem and a second objective is to show the potential capabilities of the integral approach in general. In this approach, the equations are integrated over the thickness of the concentration boundary layer thereby reducing the number of independent variables by one. Results of the present model are compared to those obtained with Brighton's model in Figs. 2 and 3. Brighton's model is considered as a reference here since it has been validated by comparison with carefully compiled experimental data. Fig. 2 is related to the hydrodynamically smooth case with the Schmidt number as parameter and Fig. 3 to the hydrodynamically rough case with $Re_o^{1/2} Sc$ as parameter. The good agreement between the numerical solution (Appendix A, Eq. (B8), obtained by means of the Runge–Kutta procedure) and Brighton's 'exact'

solution confirms the approximations and simplifications made in the present integral model.

A feature of the integral model, not encountered in Brighton's model, is related to the approximation of the concentration profile. The chosen profile, required for the integration of the height-averaged convection–diffusion equation, does not account for all the boundary conditions correctly. Comparison with Brighton's more rigorous mathematical approach shows, however, only a minor loss in accuracy of the present approximate approach. It can be concluded that the concentration profile, used in the present integral procedure and which accounts for the laminar sublayer by means of the parameter β , is approximated adequately.

Brighton's estimate of the boundary-layer thickness is based on Batchelor's model with continuous line sources (cf. Brighton [7]). The present model, in contrast, allows the determination of the thickness of the concentration boundary layer. A new power n for the velocity profile can be derived, which is consistent with the model, yielding an improved approximation to the logarithmic profile over the thickness of the concentration boundary layer.

The approximate analytic solution differs somewhat more from Brighton's 'exact' solution. The accuracy could be improved by extending the analytic solution to higher order, but the additional effort at the expense of simplicity and physical insight is hardly justified in light of the uncertainties in the parameters defining the problem (roughness, etc.).

In conclusion, the integral approach, which includes the possibility of estimating the thickness of the concentration boundary layer, provides a more intuitive, in some ways conceptually simpler, view of the problem than a mathematically more rigorous approach. The simplicity of the results obtained by means of the present approach comes at the expense of the numerical accuracy, but the compactness of the corresponding analytical expressions (see Section 2.6) meets the requirements of producing quick and reliable estimates for safety studies.

Appendix A

A 'heuristic' representation of the two-layer model for the velocity and the concentration field, including a laminar sublayer and a turbulent logarithmic boundary layer is given in this appendix.

The fully turbulent region, at a certain distance from the surface, can be described by means of the eddy viscosity $K = \kappa u_* z$, which after integration of $\tau = \rho K \partial u / \partial z$, yields the well-known logarithmic velocity profile [8]:

$$u(z) = \frac{u_*}{\kappa} \ln \left(\frac{z}{z_0} \right), \quad (\text{A1a})$$

where z_0 is an integration constant, called roughness length. In dimensionless form:

$$u^+ = \left(\frac{u}{u_*} \right) = \frac{1}{\kappa} \ln \left(\frac{z}{z_0} \right). \quad (\text{A1b})$$

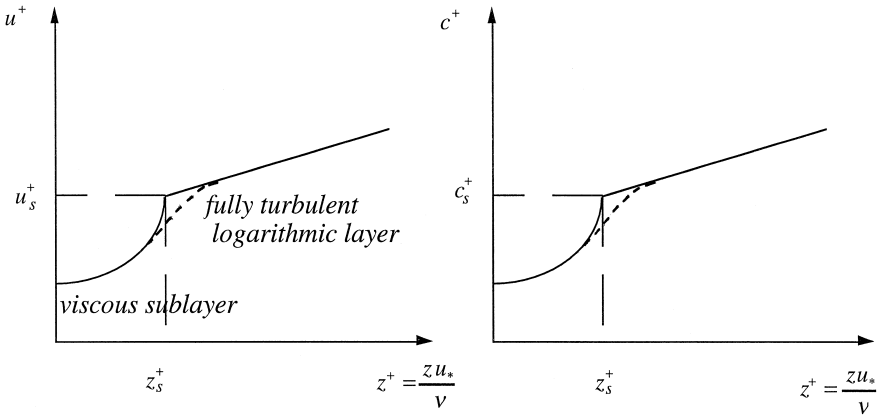


Fig. 4. Two-layer representation of the velocity field and of the concentration field.

When the mass-flow rate of the evaporating liquid is $E = -K'\partial c/\partial z$, with the eddy diffusivity K' describing the concentration field ($K' = K/\sigma$, with the turbulent Schmidt number σ), one obtains in an analogous manner a logarithmic concentration profile (see Brutsaert [1] or Schlichting [8]).

$$c_s - c = \frac{E\sigma}{\kappa u_*} \ln\left(\frac{z}{z_{ov}}\right) = \frac{E\sigma}{\kappa u_*} \left[\ln\left(\frac{z}{z_o}\right) + \ln\left(\frac{z_o}{z_{ov}}\right) \right], \tag{A2a}$$

where the integration constant z_{ov} is designated vapour roughness length. In dimensionless form:

$$c^+ = \frac{c_s - c}{(E/u_*)} = \frac{\sigma}{\kappa} \ln\left(\frac{z}{z_{ov}}\right). \tag{A2b}$$

When the Schmidt number is (close to) unity, the scaled velocity and concentration fields are similar. If the upper edge of the viscous sublayer is denoted by z_s , one obtains:

$$u_s^+ = \frac{1}{\kappa} \ln\left(\frac{z_s}{z_o}\right), \tag{A1c}$$

and:

$$c_s^+ = \frac{\sigma}{\kappa} \ln\left(\frac{z_s}{z_{ov}}\right), \tag{A2c}$$

where the subscript s denotes the quantities at the upper edge of the viscous sublayer (Fig. 4). In this simplified representation of the velocity and concentration fields, the transition layer, where both turbulent transport and molecular diffusion are present, has been neglected (dashed line in Fig. 4). The viscous sublayer is 'patched' to the fully turbulent logarithmic layer at height $z = z_s$. z_s can be eliminated from Eq. (A1c) and Eq. (A2c) to yield:

$$\ln\left(\frac{z_o}{z_{ov}}\right) = \frac{\kappa}{\sigma} (c_s^+ - \sigma u_s^+) = \frac{\kappa}{\sigma} \beta(Sc, Re_o). \tag{A3}$$

Eq. (A3) can be substituted into Eq. (A2a) with the result:

$$\frac{c}{c_s} = 1 - \frac{E}{c_s u_*} \left[\frac{\sigma}{\kappa} \ln \left(\frac{z}{z_o} \right) + \beta \right]. \tag{A4}$$

For $Sc \geq 1$, the molecular diffusion sublayer is contained inside the viscous sublayer of the velocity field. Empirical correlations, analogous to Eq. (A3), have been set up for Schmidt numbers deviating from unity. They are universal and depend on the Schmidt number only [1].

Note: Sometimes z is replaced by $(z - d_o)$, which is equivalent to a shift in reference level. The d_o is called a (zero-plane) displacement height (Brutsaert [1]).

Appendix B

The differential equation Eq. (23) and the algebraic equation Eq. (20) can be solved for the unknown dimensionless mass-flow density flux E' and the boundary-layer thickness δ_c . With the substitution:

$$\left(\frac{\delta_c}{z_1} \right) = \Gamma^{1+2n}, \tag{B1}$$

Eq. (23) can be written:

$$\frac{1}{C_1} \frac{d}{dx'} [E'\Gamma] = E', \tag{B2}$$

and Eq. (20):

$$E' = \left[\frac{\sigma}{\kappa n} \Gamma^{\frac{n}{1+2n}} + \beta \right]^{-1}. \tag{B3}$$

E' of Eq. (B3) can be substituted into Eq. (B2) to obtain:

$$\frac{1}{C_1} \frac{d}{dx'} \left\{ \Gamma \left[\frac{\sigma}{\kappa n} \Gamma^{\frac{n}{1+2n}} + \beta \right]^{-1} \right\} = \left[\frac{\sigma}{\kappa n} \Gamma^{\frac{n}{1+2n}} + \beta \right]^{-1}. \tag{B4}$$

The differentiation on the LHS of Eq. (B4) is performed and the dependent variable Γ and the independent variable x' are separated to give:

$$\frac{1+n}{1+2n} (\gamma\Gamma)^{\frac{n}{1+2n}} + 1}{(\gamma\Gamma)^{\frac{n}{1+2n}} + 1} d(\gamma\Gamma) = C_1 d(\gamma x') \text{ with } \gamma = \left(\frac{\sigma}{\kappa n \beta} \right)^{\frac{1+2n}{n}}. \tag{B5}$$

The equation can be integrated starting at $x' = 0$ (the beginning of the concentration boundary layer) with $\delta_c(x = 0) = 0$, i.e., $\Gamma(x' = 0) = 0$, or $(\gamma x') = 0$: $(\gamma\Gamma) = 0$.

An approximate analytic solution can be obtained because the integrand in Eq. (B5) is close to unity in all cases.

1. In the limit $\Gamma \rightarrow 0$, the integrand in Eq. (B5) is unity and the solution is $\Gamma = C_1 x'$.
2. In the limit $\Gamma \rightarrow \infty$, the integrand in Eq. (B5) tends towards $(1+n)/(1+2n)$ and the solution is $\Gamma = C_1 [1 + n/(1+n)]x'$.

The solution of interest lies between these two extremes. The average of the two preceding extreme conditions can be considered as an approximation:

$$\Gamma = C_1 \left[1 + \frac{1}{2} \frac{n}{1+n} \right] x', \quad (\text{B6})$$

valid for the whole range of Γ .

This solution is confirmed by a perturbation analysis where $(\gamma\Gamma)$ is expanded with respect to the small parameter $\varepsilon = n/(1+2n)$. The difference between the resulting expression for Γ and Eq. (B6) is small of order n^2 .

It has been confirmed by numerical integration of Eq. (B5), for two representative values of n (i.e., $n = 1/7$ and $n = 1/3$) that the relationship is almost linear for the range of x' of interest ($0 \leq x' < L/z_0$). The thickness of the concentration boundary layer can be obtained when Eq. (B6) is substituted into Eq. (B1):

$$\left(\frac{\delta_c}{z_1} \right) = (C_1 C_2 x')^{\frac{1}{1+2n}}. \quad (\text{B7})$$

Eq. (B5) can be written in a form suitable for a *numerical integration* (Runge–Kutta):

$$\frac{d\Gamma}{dx'} = C_1 \frac{(\gamma\Gamma)^{\frac{n}{1+2n}} + 1}{\frac{1+n}{1+2n} (\gamma\Gamma)^{\frac{n}{1+2n}} + 1}. \quad (\text{B8})$$

The integration is performed with respect to the independent variable x' because its range is well defined, i.e., $x' = 0 \div L$. By integration, one obtains $\Gamma(x')$ and in particular $\Gamma(L)$. Eq. (B3) yields $E'(L)$.

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